

Restrictions on the Effects of Preference

Variables in the Rotterdam Model

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Abstract

This study examines imposing and testing restrictions on preference variables in the Rotterdam model through the impacts of these variables on marginal utilities. An empirical analysis of the impact of a female labor force participation variable in a Rotterdam demand system for fresh fruit illustrates the methodology. This variable was modeled through its impact on marginal utilities via “adjusted” prices, following theoretical work by Basmann and Barten, among others. Results show that the female labor participation has negatively impacted the demands for citrus, while positively impacting the demands for other fresh fruit.

Key words: demand, demographic, fresh fruit, Rotterdam model.

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Introduction

Empirical studies of demand have found preference variables, along with prices and income, to be important determinants of demand. Preferences have been conditioned on various demographic variables, past consumption, advertising, and household composition variables (e.g., Barten 1964b; Philips; Deaton and Muellbauer; Theil 1980a; Hanemann 1982, 1984; Selvanathan; Pollak and Wales).

Based on the consumer's budget constraint, the effects of preference variables, income, and prices obey adding-up restrictions. Theory indicates that the effects of prices further obey homogeneity, symmetry and negativity restrictions. These conditions are referred to as general demand restrictions (Phlips).

Additional restrictions, referred to as specific restrictions in this paper¹, have also been placed on demand functions. Examples of specific restrictions are those on price effects resulting from separability (e.g., Deaton and Muellbauer; Theil 1976), and those on preference variables suggested by Theil in the context of advertising (1980a).

In this paper, specific restrictions on preference variable effects are considered in the context of the differential demand system or Rotterdam model (Theil 1971, 1975, 1976, 1980a,b). Rotterdam model coefficients for preference variables (e.g., Theil 1980a; Duffy 1987) are related back to the utility function to analyze restrictions on these coefficients.² An approach to testing specific restrictions on

preference variables is proposed, and a study of the effects of a demographic variable, the female labor force participation rate, on the demand for fresh fruit is made to illustrate the approach.

Preference variable effects are specified through a fundamental relationship between price effects and preference variable effects on marginal utilities (Ichimura; Tintner; Basmann; Barten, 1977). This relationship is singular and a specification to deal with this situation is suggested. A feature of restricting preference variable effects through this relationship is that the adding-up condition of demand is maintained, in contrast to some specifications where restrictions may be inconsistent with adding up (Bewley).

Theoretical Model

Effects of Income, Prices and Preference Variables

Our specification of how preference variables impact demand is based on Barten's (1977) fundamental matrix equation of consumer demand and follows the approach used in modeling advertising effects in the Rotterdam model by Theil (1980a); Duffy (1987, 1989), and Brown and Lee (1997) among others. Early theoretical work related to this approach was done by Basmann, Tintner, and Ichimura.

Consider the traditional consumer problem of choosing that bundle of goods that maximizes utility, subject to a budget constraint. Along with quantities of the goods in questions, one preference variable is included in the utility function. The results for this variable generalize straightforwardly to other preference variables. Formally, the consumer problem can be written as maximization of $u = u(q,$

z) subject to $p'q = x$, where u is utility; $p' = (p_1, \dots, p_n)$ and $q' = (q_1, \dots, q_n)$ are price and quantity vectors with p_i and q_i being the price and quantity of good i , respectively; x is total expenditures or income; and z is the preference variable (in general, z could be a vector of variables). The first-order conditions for this problem are $\partial u / \partial q = \lambda p$ and $p'q = x$, where λ is the Lagrange multiplier which is equal to $\partial u / \partial x$ or the marginal utility of income. The solution to the first-order conditions is the set of demand equations $q = q(p, x, z)$ and the Lagrange multiplier equation $\lambda = \lambda(p, x, z)$. The Rotterdam model is an approximation of this set of demand equations, and the demand model developed in this paper is a variant of this approximation. Analysis by Barnett, Byron and Mountain show the Rotterdam model is comparable to other popular flexible functional demand specifications like the Almost Ideal Demand System (Deaton and Muellbauer).

A fundamental relationship exists between the effects on demand of our preference variable, prices and income. We review this relationship here, as the results are required for our particular model specification. Consider the total differential of the first order conditions of the utility maximization problem, which can be written as

$$(1a) \quad U dq - p d\lambda = \lambda dp - V dz$$

$$(1b) \quad p' dq = dx - q' dp,$$

where $U = [\partial^2 u / \partial q_i \partial q_j]$ and $V = [\partial^2 u / \partial q_i \partial z]$. U is the Hessian matrix, and V is a matrix indicating how preference variable z effects the marginal utilities. Results (1a) and (1b) form a system of equations known as the fundamental matrix equation of consumer demand theory (Barten 1977).

Our particular specification of the Rotterdam model can be directly derived from fundamental

matrix equation (1). Key steps in this derivation are shown below. First, multiply (1a) by U^{-1} and rearrange to obtain

$$(2) \quad dq = U^{-1}p' d\theta + \theta U^{-1} (dp - Vdz/\theta).$$

Result (2) provides a preview of a basic relationship between the effects of prices and the preference variable. This result can be viewed as a partial demand system with the second term on the right-hand side showing the effects of prices and the preference variable, given income compensations to hold both real income and the marginal utility of income (θ) constant. The term θU^{-1} , known as the system's specific price effect (e.g., Theil 1975), is common to both price and preference variable effects. We will focus closely on this commonality in developing our model.

To obtain a total relationship demand, solve (1) and (2) for $d\theta$, substitute this solution into (2) and rearrange to find the effects of prices, income and the preference variable on demand--- $\partial q/\partial p'$, $\partial q/\partial x$ and $\partial q/\partial z'$.³ We express these results below as Hicksian or income-compensated demand equations, that is,

$$(3) \quad dq = \partial q/\partial x (dx - q' dp) + S (dp - Vdz/\theta),$$

where $\partial q/\partial x = U^{-1}p/p'U^{-1}p$, $\partial\theta/\partial x = 1/p'U^{-1}p$, and $S = \theta U^{-1} - (\partial q/\partial x) (\partial q/\partial x)' (\theta/\partial\theta/\partial x)$. The term $(dx - q' dp)$ is real income, compensated price effects are indicated by S (known as the price substitution matrix), and uncompensated price effects, $\partial q/\partial p'$, are $S - (\partial q/\partial x)q'$. The effects of the preference variable, $\partial q/\partial z'$, are $-SV/\theta$. For early formulation of $\partial q/\partial z'$, see Basmann, Tintner and Ichimura; for reviews see Phelps and Barten (1977).

Rotterdam Model

The Rotterdam model is compensated demand (3) expressed in log changes.⁴ Following Barten (1964) and Theil (1975, 1976, 1980a,b), the i^{th} demand equation for the Rotterdam model can be written as

$$(4) \quad w_i d(\log q_i) = \alpha_i d(\log Q) + \sum_j p_{ij} d(\log p_j) + \beta_i d(\log z) \quad i=1, \dots, n,$$

where $w_i = p_i q_i / x$ is the budget share for good i ; $\alpha_i = p_i (\partial q_i / \partial x)$ is the marginal propensity to consume; $d(\log Q) = \sum w_i d(\log q_i)$ is the Divisia volume index⁵; $p_{ij} = (p_i p_j / x) s_{ij}$ is the Slutsky coefficient, with $s_{ij} = (\partial q_i / \partial p_j + q_j \partial q_i / \partial x)$ being the $(i,j)^{\text{th}}$ element of the substitution matrix S ; and $\beta_i = w_i (\partial \log q_i / \partial z)$ is the preference variable coefficient.

The general restrictions on demand are (e.g., Theil 1980a,b)

$$(5a) \quad \text{adding up:} \quad \sum_i \alpha_i = 1; \quad \sum_i p_{ij} = 0; \quad \sum_i \beta_i = 0;$$

$$(5b) \quad \text{homogeneity:} \quad \sum_j p_{ij} = 0;$$

$$(5c) \quad \text{symmetry:} \quad p_{ij} = p_{ji}.$$

Coefficients α_i and p_{ij} are usually treated as constants in estimating the Rotterdam model. The coefficient β_i can also be treated as a constant, but, for placing restrictions on preference variable effects, we consider an alternative parameterization.

Restrictions on Preference Variables

Result (5a) shows that adding-up imposes one restriction on the effects of preference variable z .

In this section, additional potential restrictions on the effects of z are considered. We use the effects of

the preference variable on marginal utilities as a source of restrictions.

From (3), we found that the effects of preference variable z on demand in terms of levels can be written as $\partial q/\partial z = - (1/\eta) S V$. In transforming result (3) to obtain the Rotterdam model, we now find that β_i can also be expressed as

$$(6a) \quad \beta_i = -\sum_h p_{ih} \eta_h, \quad i = 1, \dots, n,$$

where $\eta_h = \partial \log (\partial u / \partial q_h) / \partial \log z$, that is, η_h is the elasticity of the marginal utility of good h with respect to preference variable z . Result (6a) is the Tintner-Basmann relationship in terms of the Rotterdam parameterization (see Selvanathan or Brown and Lee 1997 for discussion of this relationship with respect to advertising effects).

Our analysis of restrictions on the effects of z will be made through the coefficients η , as opposed to the coefficients β . As shown by (6a), coefficient η_h is directly related to utility, in contrast to coefficient β_i where the effects of the η_h 's and Slutsky coefficients are combined.

In terms of matrices, (6a) can be written as

$$(6b) \quad \beta = -p \eta,$$

where $\beta = [\beta_i]$, $p = [p_{ih}]$, and $\eta = [\eta_h]$.

From (5), p is singular so equation (6b) cannot be solved for η .⁶ However, using restrictions (5), we can obtain a solution. Note that we only need to know the first $n-1$ rows of β and p , since the n^{th} row of these matrices can be determined by adding-up condition (5a). Also, only the first $n-1$ columns of p are needed, since the n^{th} column can be determined from homogeneity condition (5b). Hence, deleting the n^{th} rows from β and p , and the n^{th} column from p , we can write

$$(7a) \quad \beta^* = [p^* | -p^* \gamma] [\gamma^*, \gamma_n]'$$

or

$$(7b) \quad \beta^* = p^* [\gamma^* - \gamma_n]$$

and

$$(7c) \quad \beta_n = -\gamma \beta^* = -\gamma p^* [\gamma^* - \gamma_n]$$

where $\beta^* = (\beta_1, \dots, \beta_{n-1})'$; $p^* = [p_{ij}]$, $i, j = 1, \dots, n-1$; $\gamma^* = (\gamma_1, \dots, \gamma_{n-1})$; and γ is the unit vector.

The term $(\gamma^* - \gamma_n)$ shows how the first $n-1$ elasticities of marginal utility with respect to z differ from the elasticity for good n .

In general, p^* is nonsingular so that we can solve (7b) for $(\gamma^* - \gamma_n)$, that is,

$$(7d) \quad (\gamma^* - \gamma_n) = -p^{*-1} \beta^*.$$

Given estimates of the Rotterdam model (4), one could estimate equation (7d) and determine if restrictions on γ are statistically appropriate. Alternatively, restrictions (5) can be directly imposed on (4) and the right-hand side of (7b) can be used to express β in terms of γ , such that,

$$(8) \quad w_i d(\log q_i) = \gamma_i d(\log Q) + \sum_{j=1, \dots, n-1} p_{ij} [d(\log p_j) - d(\log p_n) - \gamma_j^n d(\log z)],$$

$$i = 1, \dots, n-1,$$

where $\gamma_j^n = \gamma_j - \gamma_n$. In contrast to treating the β_i 's as constant, as suggested above for model (4), the γ_j^n 's are treated as constants in model (8). The β_i 's play the role of reduced form coefficients, while the γ_j^n 's play the role of structural coefficients.

In demand equation (8), a change in z can be viewed as resulting in "adjusted" price changes.⁷

An adjusted price change for a product is the product's actual price change minus the change in the

product's marginal utility as a result of the change in variable z ; an increase in z may increase a product's marginal utility which in turn would decrease its adjusted price and vice versa. In equation (8), the term in the bracket following the Slutsky coefficient is the relative adjusted price change for good j --the j^{th} product's actual price change, less the impact of preference variable z on the j^{th} product's marginal utility, relative to the n^{th} product's price change, less the impact of preference variable z on the n^{th} product's marginal utility (these changes are in percentages with the Rotterdam model specified in log differences).⁸ Accordingly, equation (8) expresses demand as a function of both relative price changes ($d(\log p_j) - d(\log p_n)$) and relative marginal utility elasticity changes due to z ($\gamma_j^n d(\log z)$).

The above adjusted price interpretation also has an estimation implication. Restrictions imposed on the structural coefficients (γ_j^n 's) may yield more precise Slutsky coefficient estimates, which may be important when price variation is limited, as variation in both prices and z contribute to the estimation of these coefficients (Theil 1980a).

Notice that restrictions on γ are consistent with the adding-up condition. Pre-multiplying equation (6b) by a $1 \times n$ unit vector γ_n' yields $\gamma_n' \beta = - \gamma_n' p = 0$ for any restrictions γ since $\gamma_n' p = 0$ by adding-up condition (5a). In contrast, some restrictions on β may not be consistent with adding up (Bewley). For example, in an advertising study where z is advertising on good 1, we could not have an own-advertising effect on good 1 if there were no cross advertising effects on the other goods, that is, β_1 could not be free and $\beta_i = 0$ for $i=2, \dots, n$, as this restriction would imply $\beta_1 = 0$ based on (5a); on the other hand, γ_1 could be free and $\gamma_i = 0$ for $i=2, \dots, n$.

Several studies have imposed, but not tested, restrictions on the γ 's. A common restriction

imposed in specifying impacts of advertising on demand has been that advertising for a good affects only that good's marginal utility (e.g., Theil 1980a; Duffy 1987, 1989; and Brown and Lee 1997). For example, when z is advertising for good 1, advertising only affects the marginal utility of good 1 (β_1 unrestricted; $\beta_j = 0$ for $j=2, \dots, n$) so that

$$(9a) \quad \beta_i = -p_i \beta_1.$$

That is, advertising on good 1 only changes the adjusted price for good 1. Specification of β_j^n above shows that essentially the same result can be motivated by making a weaker assumption which allows z to affect the marginal utilities of other products. Assume that an increase in product 1's advertising has a generic effect on the marginal utilities of other goods ($\beta_j = \beta_0$, $j=2, \dots, n$) and a specific effect, as well as the generic effect, on its own marginal utility ($\beta_1 = \beta_0 + \beta_1$). These assumptions result in the following restrictions

$$(9b) \quad \beta_1^n = (\beta_0 + \beta_1) - \beta_0 = \beta_1$$

and

$$(9c) \quad \beta_j^n = \beta_0 - \beta_0 = 0, \quad j=2, \dots, n.$$

Regardless of the motivation, such restrictions may not hold empirically while less restrictive ones may.

Another example of restrictions on β are those suggested by Selvanathan in a study of advertising effects. In this study, the direct utility function was assumed to be block independent with respect to both quantities and advertising. For example, suppose there are two groups, A and B, with goods 1, ..., m in group A and goods $m+1, \dots, n$ in group B, and let z be advertising on good 1 in group A. Under block independence, the utility function can be written as $u = u_A(q_A, z) + u_B(q_B)$,

where u_A and u_B and q_A and q_B are subgroup utility functions and quantity vectors, respectively. In this case, $\gamma_j^n = \gamma_j$ for $j \in A$ and $\gamma_j^n = \gamma_j = 0$ for $j \in B$. Like the case of generic and specific advertising effects in equation (9b), note that block independence need not be assumed to obtain these restrictions. Assume that good 1 advertising has generic effects on goods in group B ($\gamma_j = \gamma_0$ for $j \in B$), resulting in $\gamma_j^n = \gamma_0 - \gamma_0 = 0$ for $j \in B$; and assume that good 1 advertising has specific effects on goods in groups A (γ_j for $j \in A$), resulting in $\gamma_j^n = \gamma_j - \gamma_0$ for $j \in A$. That is, these assumptions result in essentially the same restrictions as block independence.

Brown and Lee (1997) used generic and specific restrictions in a Rotterdam model to account for generic and brand advertising effects. Generic advertising for a group of goods affected the adjusted prices of those goods in the group while brand advertising for a good affected the adjusted price of that good only.

In summary, a number of studies have imposed restrictions on preference variables in the Rotterdam model through γ instead of β . This approach allows the restrictions to be directly related to utility and preserves the adding-up condition, which may be helpful in rationalizing the specification. In previous studies, restrictions on γ have been implicitly considered as part of the maintained hypothesis. However, the foregoing results suggest that before accepting these restrictions they might be examined against an unrestricted specification. Restrictions on γ can be tested straightforwardly with usual statistical methods as illustrated in the next section.

Empirical Model and Data

Our empirical study focuses on how a demographic variable, the female labor force

participation rate or, for short, female labor participation (FLP), impacts the demand for fresh fruit.⁹

Following the above theoretical model, FLP is considered as an argument in the consumer utility function and resulting demand equations. Knowledge of how changes in this variable impacts demand can be helpful in understanding market behavior and in developing marketing strategies.

Demand models (4) and (8) were applied to annual data on per capita fresh table fruit consumption and retail prices, reported in the *Fruit and Tree Nuts, Situation and Outlook Yearbook, October 1999*, published by the United States Department of Agriculture (USDA). The period from 1980 through 1998 was studied; prices for the period before 1980 were not reported. Retail price data for table fruit were only reported for oranges, grapefruit, apples, pears, bananas and grapes.¹⁰

Reported retail orange prices were for the navel and Valencia varieties; these two price series were used to construct a weighted average retail orange price with the weights based on fresh utilization levels for navels and Valencias reported by the Florida Agricultural Statistics Service in various issues of *Citrus Summary*. Apple and pear prices were highly correlated and these two types of fruit were combined into one group. The number of fresh fruit categories studied was then five -- oranges, grapefruit, apples/pears, bananas and grapes. Mean budget shares for these categories were .18 for oranges, .07 for grapefruit, .35 for apples/pears, .22 for bananas, and .18 for grapes.

Data on the FLP were obtained from the Department of Labor, Bureau of Labor Statistics. FLP has increased from 51.5% in 1980 to 59.8% in 1998. However, changes in FLP vary over time, and this variable does not follow a simple time trend.

Application

The group of five fresh fruit categories discussed above was treated as separable from other goods. Hence, the system is conditional on expenditure allocated to the fresh fruit studied. Based on the theory of rational random behavior, the conditional real income variable (Divisia volume index) was treated as independent of the error term for each fresh fruit demand equation (Theil 1975, 1976, 1980b; Brown, Behr and Lee). The infinitely small changes implied by model (4) were measured by discrete changes as suggested by Theil (1975). The model was estimated by the maximum likelihood method obtained by iterating the seemingly unrelated regression method. As the data add up by construction, the error covariance matrix was singular and an arbitrary equation was excluded (Barten, 1969); the parameters for the excluded equation can be obtained using conditions (5) or by re-estimating the model omitting a different equation. We treat model (4) or (8), with general demand restrictions (5) imposed, as our maintained hypothesis (Keuzenkamp and Barten).

Estimates of (4) are shown in Table 1. All (conditional) marginal-propensity-to-consume estimates were positive, with three being statistically different from zero to the extent that they are twice or greater than their asymptotic standard error estimates. The estimates for grapefruit and bananas were insignificant. All (conditional) estimated own-Slutsky coefficients were negative and significant as expected based on demand theory. The cross-Slutsky coefficient estimates were either positive and significant, indicating substitution relationships, or not statistically different from zero.

Estimates of reduced form coefficients β^* in Table 1 also show that FLP positively affected the demands for bananas and grapes, while negatively affecting the demands for other fresh fruit. Exclusion

of the FLP from the model is rejected at the 10% level of significance based on the likelihood ratio test between the unrestricted model including the FLP (Table 1) versus the restricted model excluding the FLP.¹¹ The estimates for bananas, grapes and oranges were twice or greater than their asymptotic standard error estimates while those for grapefruit and apples/pears were not. Estimates of structural coefficients β^n , shown in the table, further indicate how FLP affects the marginal utilities of the different fruit. The last column of the table shows estimates of (7d), obtained directly through estimation of (8), with the n^{th} or base elasticity of marginal utility with respect to FLP being for the grape category (the estimate in the table for a given fresh fruit is that fresh fruit's elasticity of marginal utility with respect to FLP minus fresh grapes' elasticity of marginal utility with respect to FLP). These estimates suggest that elasticity of marginal utility with respect to FLP for oranges was significantly less than that elasticity for grapes (the asymptotic t-statistic was -1.93), while those for grapefruit, apples/pears and bananas were not (their asymptotic t-statistics were less than 1 in absolute value); for a given percentage change in FLP, the percentage change in grapes' marginal utility was larger than the percentage change in the orange marginal utility, but not significantly different than the percentage changes in the grapefruit, apple/pear or banana marginal utilities. Based on the above observation, the elasticities of marginal utility with respect to FLP for grapefruit, apples/pears, bananas, and grapes were assumed to be the same (structural coefficients β^n or elasticity differences, as defined in Table 1, for grapefruit, apples/pears and bananas were set to zero) while the elasticity difference for oranges was free. Based on the likelihood ratio test (Table 2), this set of three restrictions was accepted with a chi-square p-value of .17. (Thus one preference variable coefficient β_1^n ($i=1$ for oranges) is included in the model, in

contrast to including say just β_1 which would be inconsistent with adding up.)

Estimates of model (8) under the above restrictions are shown in Table 2. Generally, many of the coefficients estimates in Tables 2 are similar to the corresponding estimates in Table 1, as expected given the likelihood ratio test result. The restricted model suggests that the difference in the elasticities of marginal utility with respect to FLP for oranges and grapes is not as great as indicated by the unrestricted model. With FLP only affecting the adjusted relative price for oranges, a smaller difference (β_1^n) explains the impact of FLP through the Slutsky coefficients ($-p_{i1} \beta_1^n d(\log z)$); the second to the last column of Table 2 shows estimates of the FLP reduced form coefficients ($\beta_i = -p_{i1} \beta_1^n$).

Demand elasticities estimated at sample mean budget shares¹² are shown in Table 3. The price elasticities are uncompensated. Elasticity formulas are provided in Duffy (1987), and Brown and Lee, 1993, among others. The (conditional) expenditure elasticities ranged from .40 for bananas to 1.75 for oranges. The (conditional) own-price elasticities ranged from around -.5 for apples/pears, bananas and grapes to -.67 for oranges and -1.11 for grapefruit. The cross-price elasticities were mixed in sign, ranging from -.48 to .64, with 11 out of 20 of the estimates being insignificant. Lastly, the elasticities of demand with respect to FLP were negative for oranges and grapefruit, and positive for the other fruit, although only the elasticities for oranges and grapes were significantly different than zero. This result suggests that females in the labor force have a preference for apples/pears, bananas and grapes over oranges and, possibly, grapefruit, perhaps due to the relatively inconvenience in peeling and sectioning citrus for consumption, as suggested by Thompson, Conklin and Dono who found similar results. The estimates of the impact of FLP on the demand for grapefruit in Tables 1,2 and 3, are negative,

supporting this interpretation, but insignificant.

Concluding Comments

This paper considers an approach to specifying the effects of preference variables in the Rotterdam model. A Rotterdam specification was developed showing how preference variables affect demand through their impacts on marginal utilities. A change in a preference variable was viewed as resulting in changes in adjusted prices which were decomposed into actual price changes minus preference-variable-induced changes in marginal utilities. Restrictions on preference variables were considered through adjusted prices by imposing restrictions on the marginal utility elasticities with respect to the preference variables.

A study of the impact of the female labor force participation rate on the demands for various fresh fruit indicates that, of the fruit studied, the FLP only significantly affected the marginal utility for oranges and this effect was negative. To the extent the FLP reflects preferences for convenience in consumption, this result suggests that some consumers may view oranges as a relatively inconvenient fruit, requiring more time and effort in peeling/sectioning for consumption.

Footnotes

1. Phelps refers to these restrictions as particular restrictions; Deaton and Muellbauer refer to these as restrictions on preferences.
2. Our approach is similar to that for analyzing price separability in the Rotterdam model; Slutsky coefficients can be traced back to the utility function allowing separability restrictions on these coefficients to be straightforwardly imposed.
3. To solve for $d\beta$, multiply (2) by p' , substitute the right hand side of (1b) for $p'dq$ into this result, and rearrange terms, that is, $p'dq = d\beta p' U^{-1} p + \beta p' U^{-1} (dp - Vdz/\beta)$ or $d\beta = [(dx - q'dp) - \beta p' U^{-1} (dp - Vdz/\beta)]/p' U^{-1} p$. Substituting this solution into (2) results in $dq = U^{-1} p [[(dx - q'dp) - \beta p' U^{-1} (dp - Vdz/\beta)]/p' U^{-1} p] + \beta U^{-1} (dp - Vdz/\beta)$, or after rearrangement (3).
4. The Rotterdam model can be found by multiplying both sides of equation (3) by \hat{p} (the symbol $\hat{\cdot}$ over a vector indicates a diagonal matrix; diagonal elements equal the elements of the vector in question; off diagonal elements equal zero) and $1/x$, pre-multiply dq by the identity matrix in the form of $\hat{q} \hat{q}^{-1}$, post-multiply q' and S by $\hat{p} \hat{p}^{-1}$ and post-multiply V by $\hat{z} \hat{z}^{-1}$, that is,

$$(\hat{p} \hat{q} / x) \hat{q}^{-1} dq = \hat{p} \partial q / \partial x (dx/x - (q' \hat{p} / x) \hat{p}^{-1} dp) + (\hat{p} S \hat{p} / x) (\hat{p}^{-1} dp - (\hat{p}^{-1} V \hat{z} / \beta) (\hat{z}^{-1} dz)).$$
 This result is expressed in terms of log changes using the relationship $da/a = d \log (a)$ for any variable a .
5. The Divisia volume index is a close approximation of $d(\log x) - \sum w_i d(\log p_i)$ in (4), as shown by Theil, 1971; $d(\log Q)$ is used instead of $d(\log x) - \sum w_i d(\log p_i)$ in (4) to insure adding-up.
6. That is, there is no unique solution for β in equation (6b), since for any assumed solution β_A , the

vector $\beta_B = \beta_A + c\beta_n$ is also a solution, where c is some scalar and β_n is a conformable unit vector, since $p_n^2 = 0$.

7. Similar adjusted or corrected prices have been suggested by Barten (1964) in context of household composition effects on demand and by Fisher and Shell in context of product quality effects.
8. Equation (8) can be written as $w_i d(\log q_i) = \beta_i d(\log Q) + \sum_j p_{ij} d(\log p^*_j)$, $i=1, \dots, n-1$, where p^*_j is the relative adjusted price $(p_j/z^j)/(p_n/z^n)$.
9. Thompson, Conklin and Dono found that a similar demographic variable, the percentage of ever-married women in the labor force with children 18 years or younger, significantly affected fresh fruit demand. Their demographic variable, as well as ours, can be interpreted as a measure of the opportunity cost of time or preference for convenience in food consumption.
10. Price data for lemons, not considered a table fruit in this study, were also reported.
11. Under the null hypothesis of the restricted model, twice the difference between the maximum logarithmic likelihood value of the unrestricted model and that value for the restricted model is asymptotically distributed as a chi-square statistic with the number of degrees of freedom being equal to the number of restrictions, four in the present case.
12. The Rotterdam coefficient for general explanatory variable y is $w_i (\partial \log q_i / \partial \log y)$; hence, the elasticity formulas are based on division of the Rotterdam coefficients by the budget shares.

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Table 1. Maximum Likelihood Estimates of Unrestricted Model (a), U.S. Demand for Fresh Fruits, 1980 through 1998.

Fresh Fruit	MPC (b)	Slutsky Coefficient					FLP	E(i)-E(5) (c)
		Oranges	Grapefruit	Apples/ Pears	Bananas	Grapes		
Oranges	0.317 (0.065)	-0.065 (0.012)	-0.004 (0.007)	0.036 (0.015)	-0.006 (0.009)	0.039 (0.012)	-0.602 (0.280)	-16.602 (8.559)
Grapefruit	0.037 (0.037)	-0.004 (0.007)	-0.067 (0.013)	0.023 (0.015)	0.037 (0.012)	0.010 (0.013)	-0.095 (0.147)	-6.776 (12.518)
Apples/Pears	0.429 (0.084)	0.036 (0.015)	0.023 (0.015)	-0.086 (0.038)	0.038 (0.020)	-0.011 (0.027)	-0.379 (0.353)	-14.179 (15.386)
Bananas	0.029 (0.047)	-0.006 (0.009)	0.037 (0.012)	0.038 (0.020)	-0.096 (0.021)	0.027 (0.017)	0.461 (0.194)	-2.332 (10.408)
Grapes	0.188 (0.071)	0.039 (0.012)	0.010 (0.013)	-0.011 (0.027)	0.027 (0.017)	-0.064 (0.028)	0.615 (0.308)	
System R-square (d)					0.850			
Log of Likelihood Function					246.301			

Note: Asymptotic standard errors in parentheses.

(a) Model (4) or (8).

(b) Marginal propensity to consume.

(c) Elasticity of marginal utility of fruit with respect to FLP minus elasticity of marginal utility of grapes with respect to FLP, as defined in equation (8).

(d) Bewley (p. 42).

Table 2. Maximum Likelihood Estimates of Restricted Model (a), U.S. Demand for Fresh Fruits, 1980 through 1998.

Fresh Fruit	MPC (b)	Slutsky Coefficient					FLP	E(i)-E(5) (c)
		Oranges	Grapefruit	Apples/Pears	Bananas	Grapes		
Oranges	0.314 (0.063)	-0.064 (0.012)	-0.004 (0.005)	0.023 (0.014)	0.004 (0.009)	0.041 (0.011)	-0.537 (0.242)	-8.355 (4.164)
Grapefruit	0.028 (0.030)	-0.004 (0.005)	-0.072 (0.011)	0.023 (0.014)	0.048 (0.012)	0.005 (0.013)	-0.036 (0.046)	
Apples/Pears	0.360 (0.077)	0.023 (0.014)	0.023 (0.014)	-0.057 (0.038)	0.019 (0.022)	-0.009 (0.027)	0.195 (0.130)	
Bananas	0.089 (0.047)	0.004 (0.009)	0.048 (0.012)	0.019 (0.022)	-0.099 (0.024)	0.027 (0.019)	0.037 (0.076)	
Grapes	0.208 (0.067)	0.041 (0.011)	0.005 (0.013)	-0.009 (0.027)	0.027 (0.019)	-0.064 (0.029)	0.341 (0.170)	
System R-square (d)					0.850			
Log of Likelihood Function					243.817			
Likelihood Ratio Test Value (e)					4.968			
Degrees of Freedom (f)					3			
P-Value (g)					0.174			

Table 2. (continued)

Note: Asymptotic standard errors in parentheses.

(a) Restrictions on model (8).

(b) Marginal propensity to consume.

(c) Elasticity of marginal utility of fruit with respect to FLP minus elasticity of marginal utility of grapes with respect to FLP, as defined in equation (8).

(d) Bewley (p. 42).

(e) Twice the difference between the value of the log of the likelihood function for the unrestricted model (Table 1) and that value for the restricted model (Table 2).

(f) Number of parameters in the unrestricted model minus the number of parameters in the restricted model.

(g) Probability of obtaining likelihood ratio values that exceed the likelihood ratio test value shown in the table (right-hand tail of the chi-square distribution with three degrees of freedom).

Table 3. Conditional, Uncompensated Elasticity Estimates at Sample Means for Restricted Model (8).

Fresh Fruit	Income	Price					FLP
		Oranges	Grapefruit	Apples/Pears	Bananas	Grapes	
Oranges	1.752 (0.350)	-0.673 (0.066)	-0.140 (0.038)	-0.482 (0.160)	-0.366 (0.093)	-0.091 (0.097)	-2.998 (1.351)
Grapefruit	0.424 (0.449)	-0.141 (0.090)	-1.113 (0.162)	0.201 (0.289)	0.639 (0.212)	-0.009 (0.212)	-0.543 (0.697)
Apples/Pears	1.031 (0.221)	-0.118 (0.044)	-0.002 (0.040)	-0.524 (0.146)	-0.174 (0.077)	-0.213 (0.087)	0.558 (0.372)
Bananas	0.401 (0.211)	-0.052 (0.044)	0.191 (0.054)	-0.053 (0.132)	-0.535 (0.117)	0.048 (0.094)	0.165 (0.339)
Grapes	1.144 (0.366)	0.019 (0.072)	-0.051 (0.071)	-0.448 (0.210)	-0.107 (0.131)	-0.558 (0.174)	1.871 (0.931)

Note: Asymptotic standard errors in parentheses.