

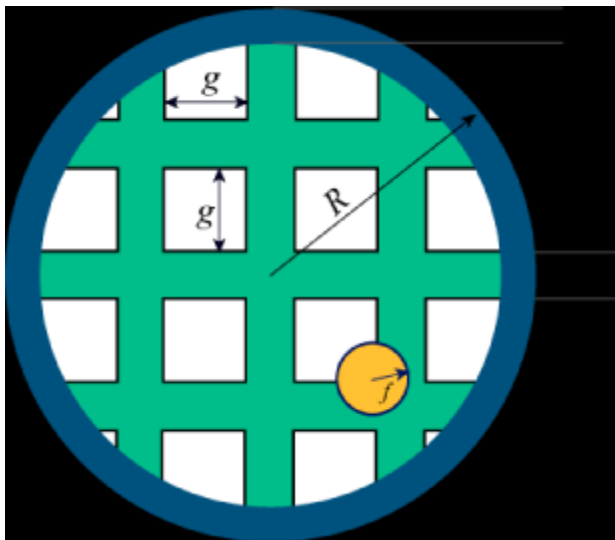
First, it should be understood that the probability of the fly being hit is a ratio between two areas.

$$\text{Probability of hitting the fly} = \frac{\text{Sum of areas that the fly is hit}}{\text{Total area}}$$

But it is easier to find the areas that the fly will NOT be hit, so we use:

$$\text{Probability of hitting the fly} = 1 - \frac{\text{Sum of areas that the fly escapes}}{\text{Total area}}$$

Total area is easy, which is: $A = \pi R^2$

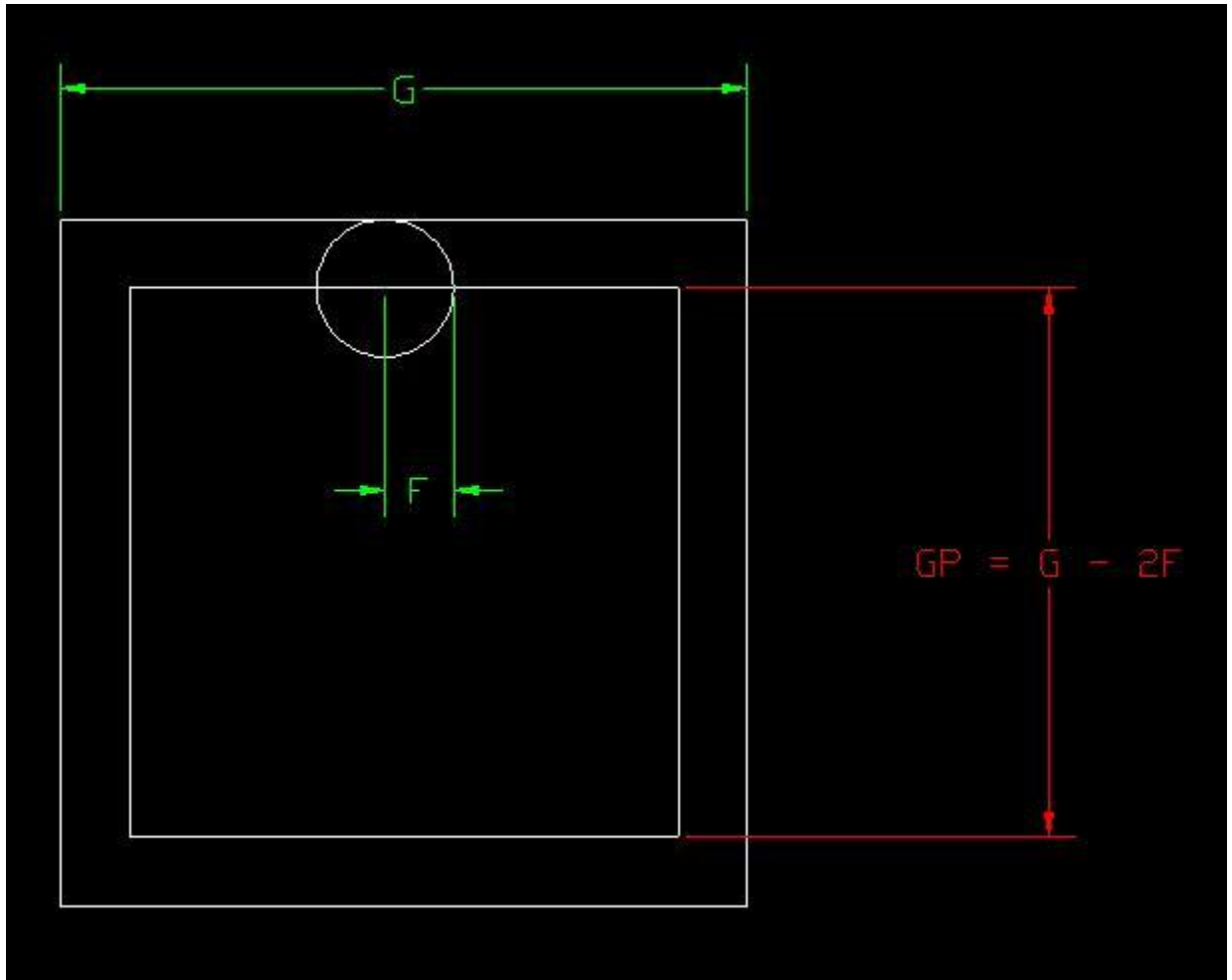


Now, we start with the harder details:

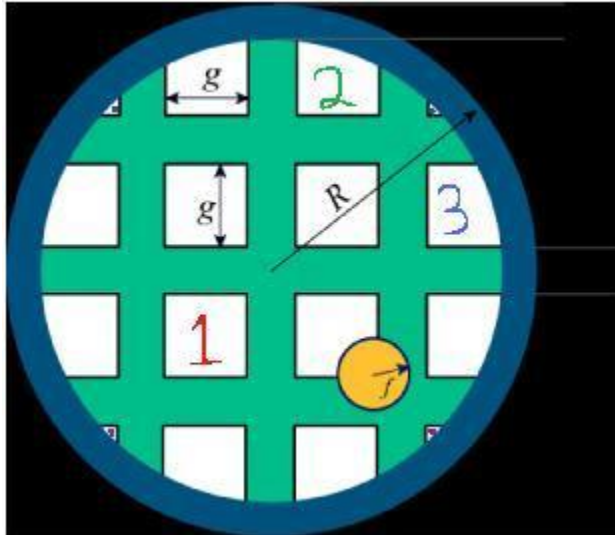
G is the input that represents the empty squares. F is the input that represents the radius of the fly.

Now for the fly NOT to be hit, the fly's CENTER must be at least F away from the edges of the square.

This smaller square will have the area of $GP \cdot GP$, where GP is the length of the side. ($GP = G - 2 \cdot F$)



As we can see, there will be several squares ($G \times G$) squares, with effective area of $(GP \times GP)$. For example, the square numbered "1" is one such square. The four squares in the middle are actually full squares.

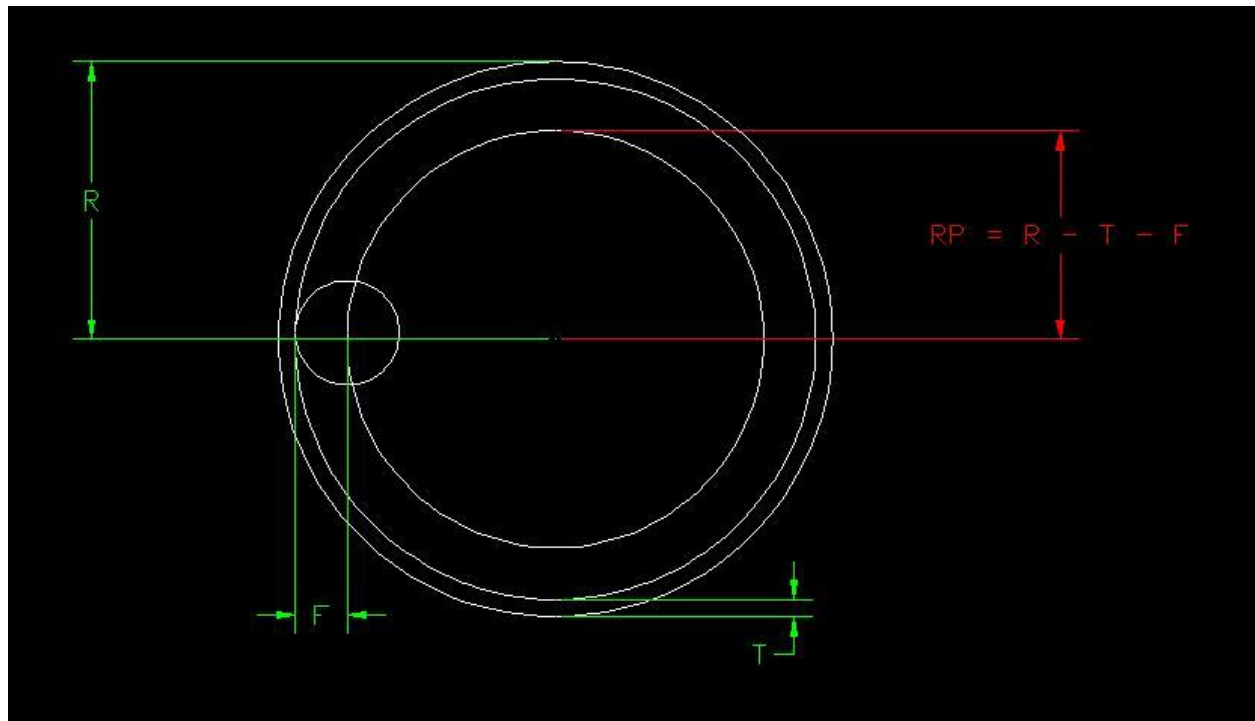


But did you notice the squares at the edges?! Like square "2" and square "3"...

Also, the four small squares at the edges. All of those fractional areas should be considered.

Another important fact is that the areas are **symmetrical horizontally, and vertically**. So you simply calculate the areas in **one quarter** of the circle and assume all other quarters have the same areas.

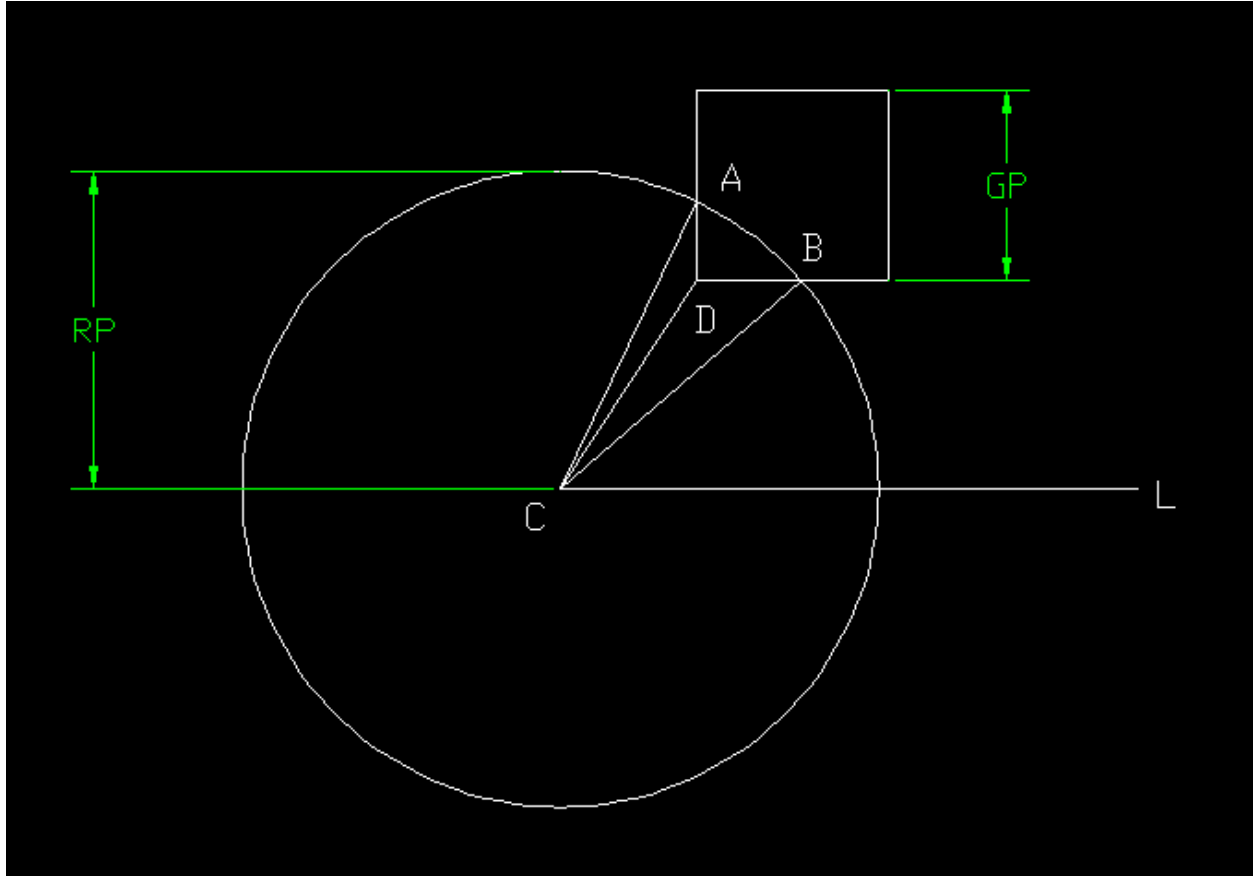
Here we define RP , which is the big outer circle of the racket. Again we subtract F for the same reason we subtracted from the square. Also, T -the thickness- should be subtracted as well.



Finally to calculate the areas of the **partial squares** we use some **geometry**:

We have four cases:

1- Circle crosses lower corner:



In this case, the

Required Area = Area of SECTOR ABC - Area of $\triangle CDB$ - Area of $\triangle CAD$

$$\text{Area of sector} = \frac{1}{2}r^2\theta$$

Where θ is the angle in radians.

Angle θ = Angle ACL - Angle BCL

If we assume C at the point of origin (0,0)

Angle ACL = $\tan^{-1} \left(\frac{A(y)}{A(x)} \right)$: A(x),A(y) are the X-Y coordinates of point A

Angle BCL = $\tan^{-1} \left(\frac{B(y)}{B(x)} \right)$: B(x),B(y) are the X-Y coordinates of point B

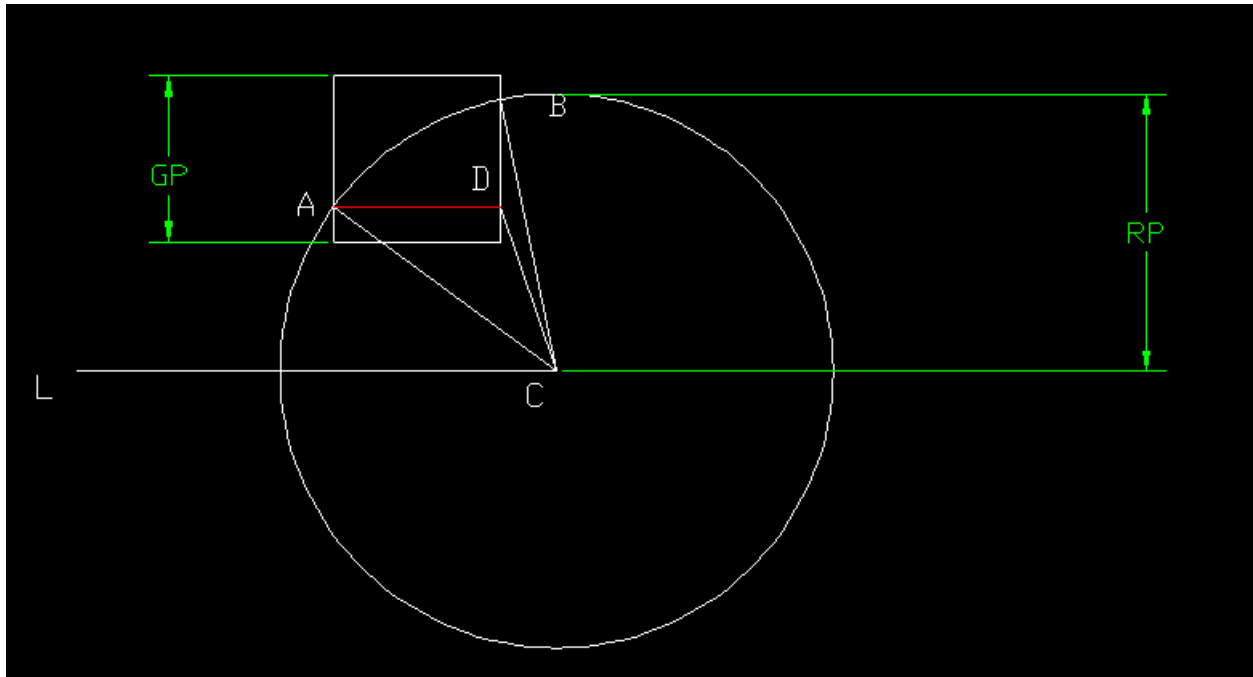
Finding the coordinates of A and B requires some algebra (which I will not discuss here). Enough to say that equation of a circle is:

$$X^2 + Y^2 = R^2$$

$$\text{Or: } Y = \sqrt{R^2 - X^2}$$

Also, to find the area of the triangles, we need to find the aforementioned coordinates of points A and B.

2- Circle crosses vertical sides:



Required Area = Area of SECTOR BCA - Area of $\triangle BCD$ - Area of $\triangle ACD$ + Area of Rectangle below AD

3- Circle crosses horizontal sides

4- Circle crosses upper corner

Cases 3 & 4 are done the same way for 1 & 2 with minor changes